Integration - Area Under A Curve

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

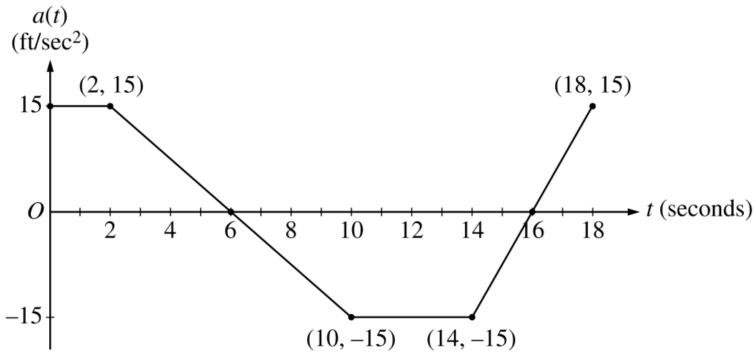
Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing , Integration - Area Under A Curve, Global or Absolute Minima and Maxima, Derivative Graphs

Paper: Part A-Calc / Series: 2001 / Difficulty: Very Hard / Question Number: 3



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

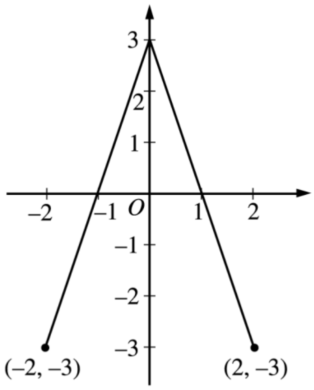
Question 2

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Derivative Graphs, Integration - Area Under A Curve, Increasing/Decreasing , Concavity, Integration Graphs

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 4



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

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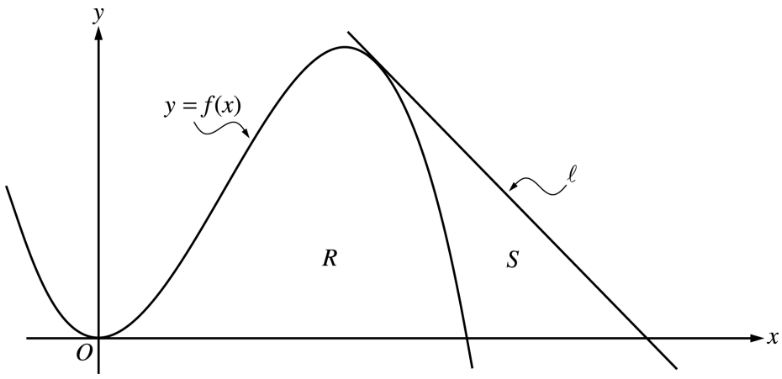
Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Tangents To Curves, Integration - Area Under A Curve, Area Between Curves, Volume of Revolution – Disc Method

Paper: Part A-Calc / Series: 2003-Form-B / Difficulty: Hard / Question Number: 1



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume of Revolution – Disc Method, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Easy / Question Number: 1

1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

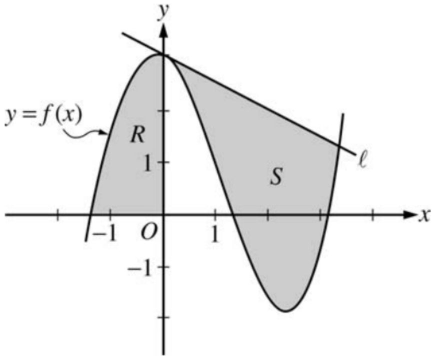
Question 5

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Integration - Area Under A Curve, Area Between Curves, Tangents To Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 1



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

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Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 1

1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Riemann Sums – Trapezoidal Rule, Average Value of a Function, Integration - Area Under A Curve, Interpreting Meaning in Applied Contexts, Modelling Situations

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Medium / Question Number: 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

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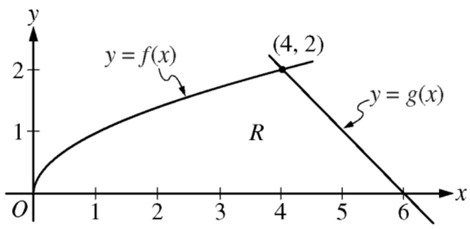
Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume using Cross Sections, Tangents To Curves

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 3



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

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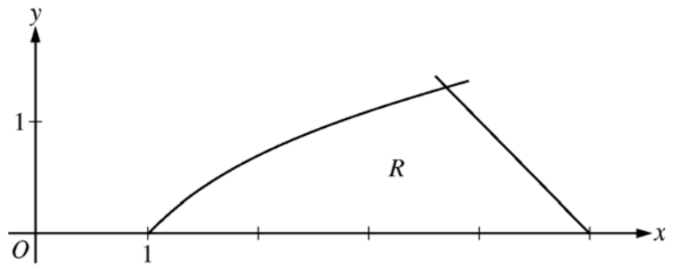
Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume using Cross Sections

Paper: Part A-Calc / Series: 2012 / Difficulty: Easy / Question Number: 2



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

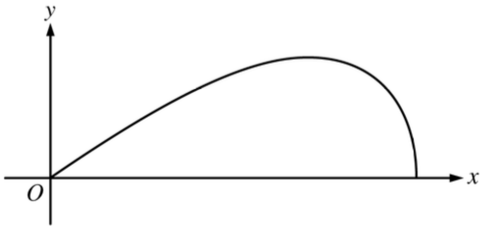
Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Integration

Subtopics: Integration - Area Under A Curve, Integration Technique – Substitution, Local or Relative Minima and Maxima, Volume of Revolution – Disc Method

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Medium / Question Number: 3



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?